[This question paper contains 6 printed pages.]

Sr. No. of Question Paper : 5007 **D** **Your Roll No.....**

Unique Paper Code : 236163

Name of the Course : **B.Sc. Mathematical Sciences**

Name of the Paper : Operational Research Concurrent : Operational Research–1

Semester : I (Mathematical Sciences)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question No. **1** is compulsory.
4. Use of non-programmable scientific calculator is permitted.

1. (i) What is Operations Research ? Suggests three advantages of Operations Research approach in decision making ?

   (ii) Explain duality in linear programming. Write the dual of the following LPP :

   Maximize $Z = 3x_1 + 4x_2 + x_3$

   Subject to $x_1 + 2x_2 + 3x_3 \leq 90$

   $2x_1 + x_2 + x_3 = 60$

   $x_1 + 2x_2 + 3x_3 \leq 90$

   $x_1, x_2 \geq 0$ and x_3 unrestricted in sign

   (iii) Consider the following LP with two variables :

Maximize $Z = 2x_1 + 3x_2$

Subject to $2x_1 + x_2 \leq 4$

$x_1 + 2x_2 \leq 5$

$x_1, x_2 \geq 0$

Determine all the basic solutions of the problem, and classify them as feasible and infeasible.

(iv) Consider the following LPP :

Maximize $Z = x_1$

Subject to $5x_1 + x_2 = 4$

$6x_1 + + x_3 = 8$

$3x_1 + + x_4 = 3$

$x_1, x_2, x_3, x_4 \geq 0$

(a) Solve the problem by inspection (do not use Gauss-Jordan row operations), and justify the answers in terms of the basic solutions of the simplex method.

(b) Repeat (a) assuming that the objective function calls for minimizing $z = x_1$.

(v) Define a convex set. Show that the following set is convex :

$C = \{(x_1, x_2) \mid x_1 \leq 2, x_2 \leq 3, x_1 \geq 0, x_2 \geq 0\}$.

(vi) A company operating 50 weeks in a year is concerned about its stocks of copper cable. This costs Rs. 240 a meter and there is a demand for 8000 meters a week. Each replenishment costs Rs. 1050 for administration and Rs. 1650 for delivery while holding costs are estimated at 25% of value held a year. Assuming no shortages are allowed, what is the optimal inventory policy for the company ?

(vii) What is inventory control ? Define the following terms associated with inventory.

(a) Holding cost

(b) Set up cost

(c) Shortage cost (5×7=35)

2. Ozark Farms uses at least 800lb of special feed daily. The special feed is a mixture of corn and soyabean meal with the following compositions.

	lb per lb of feedstuff		
Feedstuff	Protein	Fiber	Cost (Rs./lb)
Corn	.09	.02	.30
Soyabean meal	.60	.06	.90

The dietary requirements of the special feed are at least 30% protein and at most 5% fiber. Ozark farm wishes to determine the daily minimum – cost feed mix. Formulate the above as a linear programming problem and then solve it graphically. (10)

3. Solve the following LPP using two-phase method.

Minimize $z = 4x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

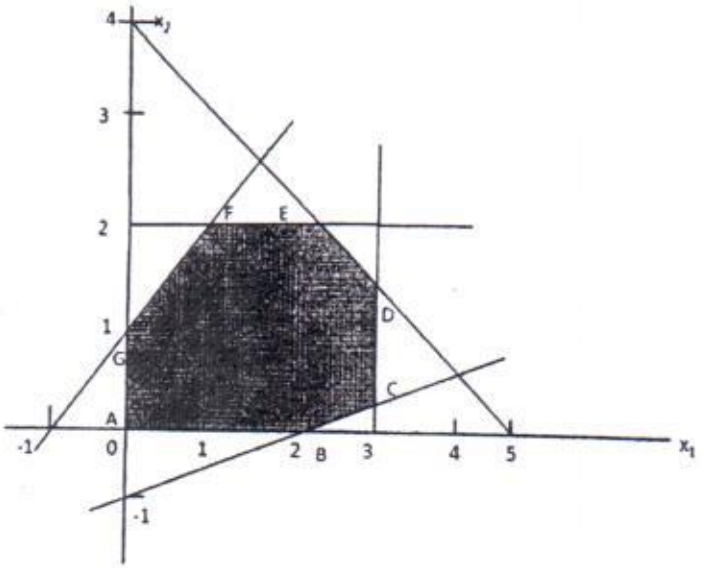
$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$ (10)

4. (a) Consider the two-dimensional solution space in figure given below. Suppose that the objective function is given as

Maximize $z = 6x_1 + 3x_2$

If the simplex iterations start at point A, identify the path to the optimum point D.



Determine the entering variable, the corresponding ratios of the feasibility condition, and the change in the value of z, assuming that the starting iteration occurs at point A and that the objective function is given as,

Maximize $z = x_1 + 4x_2$

(b) Consider the following Linear Programming Problem :

Maximize $z = 5x_1 + 2x_2 + 3x_3$

Subject to $x_1 + 5x_2 + 2x_3 = 30$

$x_1 - 5x_2 - 6x_3 \leq 40$

$x_1, x_2, x_3 \geq 0$

Given that the artificial variable x_4 and the slack variable x_5 form the starting basic variables and that M equals 100 when solving the problem, the optimal table is given in the following table. Write the associated dual problem and determine its optimal solution in two ways.

Basic	x_1	x_2	x_3	x_4	x_5	*solution*
x_1	1	5	2	1	0	30
x_5	0	−10	−8	−1	1	10
z–row	0	23	7	105	0	150

(5×2=10)

5. (a) Solve the following Linear Programming Problem using simplex algorithm and then make a comment on the nature of the solution.

Maximize $z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

(b) Consider the following Linear Programming Problem :

Minimize $z = 2x_1 + x_2$

Subject to $3x_1 + x_2 - x_3 = 3$

$4x_1 + 3x_2 - x_4 = 6$

$x_1 + 2x_2 + x_5 = 3$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

Compute the entire simplex tableau associated with the following basic solution, and check it for optimality and feasibility.

Basic variables = (x_1, x_2, x_5), Inverse = $\begin{pmatrix} \frac{3}{5} & -\frac{1}{5} & 0 \\ -\frac{4}{5} & \frac{3}{5} & 0 \\ 1 & -1 & 1 \end{pmatrix}$. (5×2=10)

6. (a) JOBCO manufactures two products on two machines. A unit of product 1 requires 2 hours on machine 1 and 1 hour on machine 2. For product 2, one unit requires 1 hour on machine 1 and 3 hours on machine 2. The revenues per unit of products 1 and 2 are Rs. 30 and Rs. 20, respectively. The total daily processing time available for each machine is 8 hours. Determine the optimal product mix. Determine the dual prices of machine 1 and machine 2 and their feasibility ranges.

(b) Describe briefly the factors affecting Inventory control. (7,3)

7. (a) The demand for a certain item is 16 units per period. Unsatisfied demand causes a shortage cost of Rs. 0.75 per unit per short period. The cost of initiating purchasing action is Rs. 15 per purchase and the holding cost is 15% of average inventory valuation per period. Item cost is Rs. 8 per unit. (Assuming that shortages are being back-ordered at the above mentioned costs). Find the minimum cost purchase quantity.

(b) Find the optimum order quantity for a product for which the price breaks are as follows :

Quantity	Unit Cost (in Rs.)
$0 \leq Q_1 < 800$	Re. 1.00
$800 \leq Q_2$	Re. 0.98

The yearly demand for the product is 1600 units per year, cost of placing an order is Rs. 5, the cost of storage is 10% per year. (5×2=10)

UNIVERSITY OF DELHI
EXAMINATION BRANCH

Please make the following announcement at the time of Examination in the following Question Paper:

Sr. No. of Question Paper : 5007 **D**

Unique Paper Code: 236163

B.Sc. (Mathematical Science)

Paper : Operational Research Concurrent: Operational Research-I

Announcement to be made:-

In Q.No.1 (ii)

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A.R. (Secrecy)